**Badger Maths**

**Problem Solving For Year 3**

Written and illustrated by Andy Seed

**Contents**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>2</td>
</tr>
<tr>
<td>National Numeracy Strategy (NNS) Links</td>
<td>3</td>
</tr>
<tr>
<td>The Problem Solving Process</td>
<td>4</td>
</tr>
<tr>
<td>Drawing a Diagram</td>
<td>5</td>
</tr>
<tr>
<td>Drawing a Table</td>
<td>19</td>
</tr>
<tr>
<td>Acting it Out or Using Concrete Material</td>
<td>34</td>
</tr>
<tr>
<td>Guessing and Checking</td>
<td>49</td>
</tr>
<tr>
<td>Creating an Organised List</td>
<td>64</td>
</tr>
<tr>
<td>Looking for a Pattern</td>
<td>77</td>
</tr>
</tbody>
</table>
Introduction

The aim of **Badger Maths Problem Solving** is to provide a valuable resource for teachers to enhance the ability of Y3 pupils in maths problem solving. The materials in the book support the National Numeracy Strategy objectives and, in particular, address the widely recognised need for a greater emphasis on using and applying mathematics.

How the book is organised

The ‘Problem Solving Process’ on page 4 outlines the essential broad strategies required for pupils at KS2 to solve problems in maths. It outlines a comprehensive four-step approach which includes spending time on explaining methods and reflecting.

The book is divided into six main sections, each dealing with problems which can be solved using a particular strategy. In each of these sections, there is a page of teaching notes about how the strategy works and three teaching examples which can be worked through with the whole class. These examples put into practice the four-step approach to problem solving outlined in the first chapter, and include answers and extension activities.

Each main section of the book includes five or six copiable problem solving task cards. Each of these pages consists of three illustrated problems to solve in the form of questions or activities. The problems, numbered 1 to 102, are presented at three broad levels to provide differentiation:

- Level A for pupils working below the national expectations.
- Level B for pupils working at the level of national expectations.
- Level C for pupils working above national expectations.

There is also a Copymaster for each section with prompt questions to help children work through the book’s four-step approach to problem solving. This may be appropriate where children need extra help or reinforcement of the appropriate strategy.

The National Numeracy Strategy (NNS) and National Curriculum

The chart opposite gives details of where each of the book’s individual numbered problems fits into the NNS framework. Most of the problems obviously come under the framework’s Problem Solving strand, although some references are included twice because they also fit into other categories such as Number Sequences or Measures.

A small number of problems do not fit easily into an NNS topic and have been included under Puzzles or Real Life Problems.

There are no references to the Calculations strand of the NNS since the book is primarily concerned with using and applying maths rather than the mechanics of calculation, although it must be stressed that pupils should be encouraged to talk about the methods they use to solve problems at every opportunity.

The material in the book also addresses the Using and Applying/Problem Solving strands of each attainment target in the National Curriculum for maths at KS2.

Using this book

The book is flexibly structured and designed to be used in several different ways:

- To help teachers teach effective strategies and approaches for solving problems.
- To resource individual NNS topics, such as Length or Fractions, particularly where extra practice at using and applying maths is required.
- To provide differentiated practice at various problem solving strategies, e.g. drawing tables and using diagrams.
- For homework exercises or extra preparation for other maths assessments.
<table>
<thead>
<tr>
<th>NNS Strand</th>
<th>Numeracy Framework topic</th>
<th>NNS ref</th>
<th>Problem number and page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers and the Number System</td>
<td>Properties of numbers and number sequences</td>
<td>16-21</td>
<td>7 (p13), 10 (p14), 18 (p16), 19 (p26), 22 (p27), 24 (p27), 34 (p31), 37 (p41), 42 (p42), 55 (p55), 57 (p55), 58 (p56), 59 (p56), 60 (p56), 61 (p57), 62 (p57), 66 (p58), 69 (p59), 70 (p60), 74 (p69), 78 (p70), 79 (p71), 84 (p72), 89 (p82), 92 (p83), 93 (p83), 94 (p84), 97 (p85), 98 (p85), 100 (p86)</td>
</tr>
<tr>
<td>Solving Problems</td>
<td>Making decisions</td>
<td>74-75</td>
<td>7 (p13), 14 (p15), 25 (p28), 31 (p30), 34 (p31), 37 (p41), 49 (p45), 68 (p59), 75 (p69), 76 (p70), 82 (p72), 93 (p83), 98 (p85)</td>
</tr>
<tr>
<td>Solving Problems</td>
<td>Reasoning about numbers and shapes: puzzles</td>
<td>78-79</td>
<td>1 (p11), 2 (p11), 4 (p12), 17 (p16), 22 (p27), 23 (p27), 28 (p29), 35 (p31), 39 (p41), 40 (p42), 46 (p44), 47 (p44), 50 (p45), 51 (p45), 52 (p46), 54 (p46), 55 (p55), 56 (p55), 59 (p56), 62 (p57), 63 (p57), 65 (p58), 66 (p58), 70 (p60), 72 (p60), 75 (p69), 96 (p84), 99 (p85), 100 (p86)</td>
</tr>
<tr>
<td>Real life problems</td>
<td></td>
<td>82-83</td>
<td>10 (p14), 16 (p16), 19 (p26), 27 (p28), 37 (p41), 53 (p46), 67 (p59), 76 (p70), 77 (p70), 80 (p71), 81 (p71), 82 (p72), 83 (p72), 84 (p72), 85 (p73), 86 (73)</td>
</tr>
<tr>
<td>Problems involving money</td>
<td></td>
<td>84-85</td>
<td>18 (p16), 26 (p28), 68 (p59), 71 (p60), 87 (p73), 102 (p86)</td>
</tr>
<tr>
<td>Problems involving measures</td>
<td></td>
<td>86-87</td>
<td>8 (p13), 11 (p14), 12 (p14), 13 (p15), 16 (p16), 21 (p26), 22 (p27), 67 (p59), 101 (p86)</td>
</tr>
<tr>
<td>Problems involving time</td>
<td></td>
<td>88-89</td>
<td>15 (p15), 20 (p26), 45 (p43)</td>
</tr>
<tr>
<td>Measures, Shape and Space</td>
<td>Length, mass and capacity</td>
<td>90-95</td>
<td>8 (p12), 11 (p14), 12 (p14), 13 (p15), 16 (p16), 21 (p26), 41 (p42)</td>
</tr>
<tr>
<td>Measures, Shape and Space</td>
<td>Properties of 3-D and 2-D shapes</td>
<td>102-105</td>
<td>4 (p12), 17 (p16), 52 (p46)</td>
</tr>
<tr>
<td>Measures, Shape and Space</td>
<td>Position and direction</td>
<td>108-109</td>
<td>3 (p11), 5 (p12), 6 (p12), 8 (p13), 9 (p13), 14 (p15), 37 (p41), 38 (p41), 43 (p43), 47 (p44), 73 (p69), 88 (p82), 90 (p82), 91 (p83)</td>
</tr>
<tr>
<td>Handling Data</td>
<td>Organising and interpreting data</td>
<td>114-117</td>
<td>20 (p26), 21 (p26), 22 (p27), 23 (p27), 24 (p27), 25 (p28), 26 (p28), 27 (p28), 28 (p29), 29 (p29), 30 (p29), 31 (p30), 32 (p30), 33 (p30), 34 (p51), 35 (p31), 36 (p31)</td>
</tr>
</tbody>
</table>
Problem Solving

THE PROBLEM SOLVING PROCESS

It is important that pupils follow a logical and systematic approach to their problem solving. Following these four steps will enable pupils to tackle problems in a structured and meaningful way.

STEP 1: UNDERSTANDING THE PROBLEM

• Encourage pupils to read the problem carefully a number of times until they fully understand what is wanted. They may need to discuss the problem with someone else or re-write it in their own words.
• Pupils should ask internal questions such as, what is the problem asking me to do, what information is relevant and necessary for solving the problem?
• They should underline any unfamiliar words and find out their meanings.
• They should select information they know and decide what is unknown or needs to be discovered. They should see if there is any unnecessary information.
• A sketch of the problem often helps their understanding.

STEP 2: PUPILS SHOULD DECIDE ON A STRATEGY OR PLAN

Pupils should decide how they will solve the problem by thinking about the different strategies that can be used. They could try to make predictions, or guesses, about the problem. Often these guesses result in generalisations which help to solve problems. Pupils should be discouraged from making wild guesses but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved.

Some possible strategies include:
• Drawing a sketch or table.
• Acting out situations or using concrete materials.
• Organising a list.
• Identifying a pattern and extending it.
• Guessing and checking.
• Working backwards.
• Using simpler numbers to solve the problem, then applying the same methodology to the real problem.
• Writing a number sentence.
• Using logic and clues.
• Breaking the problem into smaller parts.

STEP 3: SOLVING THE PROBLEM

• Pupils should write down their ideas as they work so that they don’t forget how they approached the problem.
• Their approach should be systematic as far as possible.
• If stuck, pupils should reread the problem and rethink their strategies.
• Pupils should be given the opportunity to orally demonstrate or explain how they reached an answer.

STEP 4: REFLECT

• Pupils should consider if their answer makes sense and if it has answered what was asked.
• Pupils should draw and write down their thinking processes, estimations and approach, as this gives them time to reflect on their practices. When they have an answer, they should explain the process to someone else.
• Pupils should ask themselves ‘what if’ to link this problem to another. This will take their exploration to a deeper level and encourage their use of logical thought processes.
• Pupils should consider if it is possible to do the problem a simpler way.
Drawing a Diagram
Teaching Notes

**Drawing a Diagram**

Drawing a picture of a word problem often reveals aspects of the problem that may not be apparent at first. If the situation described in the problem is difficult to visualise, a diagram, using simple symbols or pictures, may enable pupils to see the situation more easily. The diagram will also help pupils to keep track of the stages of a problem where there is more than one step.

In order to use the strategy of drawing a diagram effectively, pupils will need to develop the following skills and understanding.

**Using a line to symbolise an object**

Simple line drawings help pupils to visualise a situation. For example, consider the following problem: Brian drew four marks on a long stick, starting at the beginning and finishing at the end. Each mark was one metre apart. How long was the stick? In response, pupils may calculate mentally $4 \times 1 = 4$, but the stick is in fact three metres long. If pupils draw the stick and markers, they will be able to see this:

```
1m
```

**Using a distance line to display the information**

A distance line helps to show distance or movement from one point to another. Pupils were asked to calculate how far they were from one end of a 30m path, if they were 9m from the other end. In this case, drawing the line and marking the distances on it can help them to 'see' the problem:

```
  9m

30m
```

**Mapping or showing direction**

Pupils will sometimes be faced with diagrams that require them to have an understanding of direction. They will also meet problems where they are asked to plot a simple course by moving up, down, left or right on a grid. They will also need to use the compass points to direct themselves – north, south, east, west.

Pupils will also need to become familiar with measurement words which may be unfamiliar to them, such as pace. Opportunities should be given for the pupils to work out how many paces it takes to cover the length and breadth of the classroom or to pace out the playground, so they develop a means of comparison.

An ability to use a simple map is also important, as maps are sometimes used in mathematical problems such as this one:

Which is the shortest route from Bigtown to Seaby?

```
Bigtown

Loferd

Feleng

16 Km

11 Km

23 Km

Seaby
```

**Showing the relationships between things**

Pupils will find it helpful to draw diagrams and use symbols in order to visualise the relationships between things.

For example:

- Joe Brown hair
- Asif Blue eyes
- Ben Freckles

**Drawing a picture**

Drawing a picture can help pupils to organise their thoughts and so simplify a problem.

How many different rectangles can you make with four squares?
**EXAMPLE 1**

A baby Floop from Mars built a tower using four different Lugo bricks. The blue brick was below the green brick. The green brick was below the red brick. The yellow brick was at the bottom. Which brick was on top?

Then we are told that the green brick is below the red brick. The red brick can be added to the drawing:

![Diagram](image)

Finally, we know that the yellow brick is at the bottom:

![Diagram](image)

Pupils should be taught to go back to the question once the drawing is made, to check that their picture conforms to what the question says.

In this case, we can clearly see that the red brick was the one on top of the tower.

**Understanding the problem**

**What do we know?**
- We know there are four bricks in the tower.
- We know which brick was at the bottom.
- We know that some bricks were below others.

**What do we need to find out?**

**Questioning:**
- How can we find out which brick was on top?

**Planning and communicating a solution**

**What we did**

It is important that pupils develop their ability to logically explain their strategy. They should try to use mathematical language and drawings such as simple pictures, charts and diagrams in their explanation or during the problem solving process.

Some pupils will attempt to answer straight away without a diagram: some of them will be correct and some will not. Pupils should be encouraged to draw a picture so that the problem is clarified. They will then be able to see the order of the bricks.

**Step-by-step explanation**

First, we are told that the blue brick was below the green brick. This is straightforward to draw:

![Diagram](image)

Once pupils have reflected on the solution, they can generalise about problems of this type and see how this solution can be applied to similar problems. Would this method work if there were five bricks or ten? Or if the bricks were in a horizontal line? They should think about how else the problem could be solved – by using Lego bricks, perhaps. Pupils should also consider if the method can be made quicker or easier.

**Extension**

What if five bricks were used and a white brick was above the yellow? Could we still solve the problem if one of the pieces of information was missing? Do we know which brick is at the bottom if there are four bricks and the blue is above the yellow but below the red and the red is below the green?
EXAMPLE 2
A single piece of string is cut with scissors five times. How many pieces will there be?

Understanding the problem
What do we know?
We know there is one piece of string.
We know that it is cut five times.

What do we need to find out?
Questioning:
Is the answer five? If you cut a piece of string once, how many pieces do you get? Can a diagram or picture help us to answer the question?

Planning and communicating a solution
What we did
In this case a simple diagram helps to make the answer clear. First, draw a line to represent the piece of string.

Each cut can be represented by a gap or a line across the string. These can also be numbered, to show the number of cuts.

Both of these diagrams make it clear that five cuts produce six pieces.

Reflecting and generalising
Pupils who arrived at the answer five probably did not visualise the problem. This question is a good example of how a simple diagram or drawing can show a clear solution and help children to begin to see patterns. For those children who are confused by the diagram, using concrete materials may be necessary: allow them to cut a piece of string five times or use a strip of paper.

Extension
What if the string was cut six times? Nine times? 100 times? Can pupils see a pattern? It is also useful to reverse the question: how many cuts are needed to make seven pieces of string? How many cuts would give us 23 pieces? It should be emphasised throughout that the string is not doubled up at any point when it is cut. Another way to extend the question for more able pupils is to add a second factor. A piece of string is cut four times to make pieces each 6cm long. How long was the string to start with? A rope is cut into nine pieces. Each cut takes four seconds. How long will it take to cut the rope into pieces?
EXAMPLE 3
An Obble builds a robot and tells it to go to the shops which are 8km north of his house. The robot goes 2km west, then 4km north, then 2km east, then 2km north. Does it reach the shops? Draw the route on a grid to help you find the answer.

Understanding the problem
What do we know?
We know that the shops are 8km north of the house.
We know the route that the robot travels:
- 2km west
- 4km north
- 2km east
- 2km north

What do we need to find out?
Questioning:
Does the robot reach the shops 8km to the north of the house? Why will a grid help us to answer this question? Do we know what km means? How can we use the grid to help with directions?

Planning and communicating a solution
What we did
In this problem, a simple grid needs to be provided for children to draw on.

It makes sense to use each square to represent 1km. The position of the house and the shop can be marked on the grid. It’s also a good idea for pupils to show the four compass directions to help them plot the course.

The next step is to carefully mark the robot’s route on the grid. Pupils should use arrows to show direction and check their route against the question to see if it’s correct. Well shown like this, it can be clearly seen that the robot does not reach the shops, having only travelled 6km north.

Reflecting and generalising
This is quite a complex problem, with a long question containing lots of information which must be understood. Pupils who do not draw an accurate route are likely to end up guessing; the problem also relies on a knowledge of compass directions. However, once children are familiar with representing routes on a simple grid, they should have no difficulty with this type of question.

Extension
What does the robot need to do to reach the shops from his final position? Apart from simply travelling 8km north, can the pupils find 3 other routes he could have taken? These should each be written as a list and each marked on a grid. Do pupils notice anything about how far east and west he must travel to reach the shops?
Drawing a Diagram

★ Understanding the problem
List what you know from reading the problem.

★ What do you need to find out?
Are there any words you don’t know?
Is there anything you don’t understand?

★ Finding and writing the answer
Can you draw a picture or diagram to help you find the answer?
Will a line be helpful?
Can measurements or directions be shown in your drawing?
Can you use symbols like arrows or shapes?

★ Thinking about the problem
What did you find?
Did you check your answer to see if it matched the question?
Was there another way you could have done it?
**PROBLEM 1**  
**Level A**  
Pizzas on Pluto are square. If a pizza has to be cut into 6 strips, how many cuts are needed?

![Pizza and Knife](image)

**PROBLEM 2**  
**Level A**  
How many separate triangles can you make using 11 pencils?

![Box of Pencils](image)

**PROBLEM 3**  
**Level A**  
A Fimp keeps her 4 jumpers in a neat pile. The yellow one is below the blue one. The red one is above the blue one. The green one is below the yellow one. Which jumper is at the bottom of the pile?
**PROBLEM 4**  
A Zozz has put 4 square tiles together to make a larger square. He then put a line of the same tiles around the outside of the square to make a bigger square. How many tiles did he use altogether?

**PROBLEM 5**  
A Little Barp spreads out 7 gold coins in a row. She then puts a silver coin between each gold coin. How many coins are in the row altogether?

**PROBLEM 6**  
A Grunf sandwich is made of 4 slices of bread. There are 3 pieces of cheese between each slice of bread. How many pieces of cheese are needed to make a Grunf sandwich?
**Problem 7**

Phop planted 3 rows of onions, with 6 in each row. Birds ate half of the first row. How many onions were left altogether?

**Problem 8**

A Jiva-Jiva went for a walk from her house. She walked north for 2 kilometres, then west for 1 kilometre, then south for 1 kilometre, then east for 2 kilometres.

Did she finish at home?

**Problem 9**

A Brip walks up 9 steps of a staircase then goes back 4 steps to pick up something she has dropped. She then walks 8 steps to the top. How many steps does the staircase have?
**PROBLEM 10**

Ziggos make cans of rock juice to sell to other planets. They pack the cans in square boxes, 4 cans wide. How many cans does each box hold?

**PROBLEM 11**

A swimming pool is 10 metres long and 6 metres wide. Each day a Yox walks around it to check for beetles. How far does he walk each day?

**PROBLEM 12**

A Gupp is making a new garden fence. It will be made up of 12 vertical posts, each 2 metres apart. The posts will be joined by wires. How long will the fence be?
**PROBLEM 13**

The houses on planet Moobu are 3 metres apart. Each house is 10 metres wide. How long is a row of 5 houses on Moobu?

**PROBLEM 14**

A Quock chess board is a rectangle covered in squares. It has a centre square with 2 squares either side of it in one direction and 3 squares either side of it in the other direction. How many squares cover the board?

**PROBLEM 15**

On a small planet, a group of Buvvers are making fishing rods by tying sticks together. It takes 3 minutes to tie 2 sticks together. How long will it take to join 6 sticks to make a long fishing rod?
**Problem 16**  
**Measures**  
Glob, Glab, Glub and Gleb are four aliens waiting in a long queue. Glob is at the front. Glab is 6m behind him. Glub is 18m behind Glob. Gleb is 20m behind Glub. How far apart are Glab and Gleb?

**Problem 17**  
**Shape & Space**  
A Slimp arranged 15 matchsticks to make 5 squares like this one. How did he do it?

**Problem 18**  
**Numbers 123**  
On the planet Mynn, the unit of money is called the fip. It is planned to plant a line of 14 trees on Mynn, costing 52 fips in total. The line starts with a noggo tree and every third tree is also a noggo tree, costing 5 fips. How much does each of the other trees cost?
Answers to Task Cards

**Drawing a Diagram**

**PROBLEM 1**
5 cuts are needed to cut the pizza into six strips.

![Pizza diagram](image)

**PROBLEM 2**
It is possible to make 3 triangles with 11 pencils.

![Triangles diagram](image)

**PROBLEM 3**
The green jumper is at the bottom of the pile.

![Jumper diagram](image)

**PROBLEM 4**
The Zozz used 16 tiles in total.

![Tiles diagram](image)

4 x 4 = 16

**PROBLEM 5**
There are 13 coins in the row.

![Coins diagram](image)

7 + 6 = 13

**PROBLEM 6**
9 pieces of cheese are needed for a Grunf sandwich.

![Cheese diagram](image)

3 x 3 = 9

**PROBLEM 7**
15 onions were left.

![Onions diagram](image)

6 + 6 + 3 = 15

**PROBLEM 8**
No, the Jiva-Jiva did not finish at home.

![Path diagram](image)

**PROBLEM 9**
The staircase has 13 steps.

![Stairs diagram](image)
**Problem 10**
Each box holds 16 cans.

\[ 4 \times 4 = 16 \]

**Problem 11**
The Yox walks 32m.

\[ 10 + 10 + 6 + 6 = 32 \]

**Problem 12**
The fence will be 22m long.

\[ 11 \times 2 = 22 \]

**Problem 13**
A row of five houses is 62m long.

\[ 5 \times 10 = 50 \]
\[ 4 \times 3 = 12 \]
\[ 50 + 12 = 62 \]

**Problem 14**
A Quock chessboard has 35 squares.

\[ 7 \times 5 = 35 \]

**Problem 15**
It will take 15 minutes to make a fishing rod with six sticks.

5 joins are needed: \[ 5 \times 3 = 15 \]

**Problem 16**
Glab and Gleb are 32m apart.

\[ 20 + 12 = 32 \]

**Problem 17**
(or an equivalent arrangement)

**Problem 18**
The other trees cost 3 fips each.

The noggos cost \[ 5 \times 5 = 25 \] fips
\[ 52 - 25 = 27 \] so the other trees cost 27 fips in total.
There are 9 other trees: \[ 27 \div 9 = 3 \]